EFFECT OF TEMPERATURE PERTURBATIONS ON PLASMA STABILITY

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1. The theory of the stability of magnetic confinement of plasma has been developed by allowing for the effect of dissipative factors on unstable oscillations [1,2]. This led to the discovery of a number of dangerous instabilities that are absent in a medium without dissipation. Allowance for electron-ion friction in an inhomogeneous plasma [3] gave rise to an instability (so-called drift-dissipative) that can cause anomalous Bohm diffusion [4] with the coefficient

$$D \sim cT_0 / eH_0, \tag{1.1}$$

where c is the speed of light in vacuum, H_0 is the magnetic-field strength, T_0 is the plasma temperature, and e is the electron charge.

This result explained a number of experiments [2], although the theory of drift-dissipative instability does not include all the cases in which diffusion is observed [5].

Ion currents transverse to H_0 (in particular, inertial ion drift) play a vital role in the development of drift-dissipative instability. Here longitudinal ion motion is not considered. At the same time, an increase in $k_z/k_\perp$ (k_z is the projection of the wave vector k onto the magnetic-field direction and $k_\perp$ is the projection of k in the perpendicular direction) can change the situation, and longitudinal ion motion can be more significant. This is especially true in the frequency range $\omega \leqslant k_z v_{Ti}$ (v_{Ti} is the ion thermal velocity). But, in general, we are discussing long-wave perturbations in a direction perpendicular to H_0, i.e., perturbations that can result in great diffusion when instability develops.

The study of plasma stability in the frequency range $\omega \leqslant k_z v_{Ti}$ is important in connection with the efficiency of installations with crossed field lines [6].

It should be noted that increased plasma stability in this frequency range has been assumed by a number of authors (see [7], for example).

The part of the electron-ion friction which depends on the mean relative velocity between electrons and ions, i.e., on the current density j, is responsible for the drift-dissipative instability in an inhomogeneous plasma. Electron-ion friction also arises when an electron-temperature gradient (thermal force) is present [8]. As is well known, thermal force arises because a smaller friction force acts on "heated" electrons than on cold electrons. The effect of thermal force on plasma stability has received little study. Since thermal force is related to the electron-temperature gradient, it is clear that its role is determined by the nature of the temperature perturbations in the plasma. The examination below shows that successive allowance for the longitudinal motion of the electron and ion gases makes it necessary to take into account perturbation of the electron and ion temperatures, even if the initial plasma temperature is homogeneous.

2. Now let us derive the dispersion equation in the approximation that interests us. We will examine the potential perturbations (rot **E** = 0; **E** is the electric field of the perturbation). Assuming that the initial parameters of the plasma vary slowly, we take the perturbations in the form

$$\sim \exp\,(i\omega t + ik_y y + ik_z z).$$

We ignore inertial ion drift and the finiteness of the Larmor ion radius, but we consider longitudinal ion motion. Then, the equation of conservation of charge takes the form

$$v_{ze} - v_{zi} = 0, \tag{2.1}$$

where v_{ze} and v_{zi} are the longitudinal perturbed electron and ion velocities, respectively.

We also need the equations

$$i\omega M v_{zi} n_0 = sik_z n_0 T_e + en_0 E_z - ik_z n_0 T_i - $$

$$- ik_z n T_0 - {}^4/_3 k_z^2 T_0 n_0 v_{zi} / \nu_{i/i}, \tag{2.2}$$

$$(1+s)\, ik_z n_0 T_e - ik_z n T_0 - en_0 E_z = 0, \tag{2.3}$$

$$i\omega n + cE_y n_0' / H_0 + ik_z v_{ze} n_0 = 0, \tag{2.4}$$

$${}^3/_2 i\omega T_e + iT_0 k_z v_{ze} = -\chi_e k_z^2 T_e - 3\nu_{i/e} (T_e - T_i), \tag{2.5}$$

$${}^3/_2 i\omega T_i + iT_0 k_z v_{zi} = -\chi_i k_z^2 T_i + 3\nu_{i/e} (T_e - T_i), \tag{2.6}$$

$$v_{xe} = c\frac{E_y}{H_0} + ik_y \frac{c}{eH_0}\left(T_e + T_0 \frac{n}{n_0}\right), \tag{2.7}$$

$$v_{xi} = c\frac{E_y}{H_0} - ik_y \frac{c}{eH_0}\left(T_i + T_0 \frac{n}{n_0}\right), \tag{2.8}$$

$$s = 0.71, \quad n_0' = \frac{dn_0}{dx}, \ \text{ and } \ \omega_i = k_y \frac{cT_0}{eH_0}\frac{n_0'}{n_0}.$$

Here, (2.2) and (2.3) are the equations of ion and electron motion along H_0, respectively; (2.4) is the electron continuity equation; (2.5) and (2.6) are, respectively, the heat-balance equations for electrons and ions; M is the ion mass; n, T_e, and T_i are the perturbations of density, and electron and ion temperature, respectively; $n_0(x)$ is the initial plasma density; T_0 is the initial plasma temperature, which is assumed to be homogeneous; χ_e and χ_i are the coefficients of electron and ion thermal conductivity along H_0 referred to a single particle; $\nu_{i/i}$ is the ion-ion collision frequency (Eq. (2.2)) incorporates a term proportional to $\nu_{i/i}^{-1}$, which arises from the longitudinal component of the ion-viscosity tensor); and $\nu_{i/e}^{-1}$ is the electron-ion heat-transfer time.* Equations (2.7) and (2.8) are, respectively, expressions for the electron and ion velocities along the inhomogeneity. The above equations also allow for thermal force, which here plays a special role in (2.2) and (2.3): these are the terms proportional to s.

Equations (2.1)–(2.8) lead to the following dispersion equation:

$$\frac{{}^3/_2\omega\,(1 - {}^4/_3 ik_z^2 v_{Ti}^2 / \nu_{i/i}\omega)}{k_z^2 v_{Ti}^2}(\omega - \omega_i)\left\{\left(\omega - \right.\right.$$

$$\left.- \frac{2}{3} i\chi_e k_z^2\right)\left(\omega - 2i\nu_{i/e} - \frac{2}{3} i\chi_i k_z^2\right) -$$

$$\left. - 2i\nu_{i/e}\omega - \frac{4}{3}\nu_{i/e}\chi_i k_z^2\right\} = 5\omega^2 +$$

$$+ \ \omega\left\{2s\omega_i - \frac{8}{3} ik_z^2\chi_e - 20i\nu_{i/e}\right\} -$$

$$- 4\nu_{i/e}\chi_e k_z^2 - \frac{4}{3}\chi_i\chi_e k_z^4 + \frac{2}{3} i\chi_e k_z^2\omega_i - 8is\omega_i\nu_{i/e}. \tag{2.9}$$

*The necessity of allowing for heat transfer between electrons and ions was pointed out to the author by B. B. Kadomtsev.

Consider the case of $\omega \ll k_z v_{T_i}$. Then, from Eq. (2.9), we have

$$5\omega^2 + \omega\left\{2s\omega_i - \frac{8}{3} ik_z^2\chi_e - 20i\nu_{i/e}\right\} - 4\nu_{i/e}\chi_e k_z^2 -$$

$$- \frac{4}{3}\chi_i\chi_e k_z^4 + \frac{2}{3} i\chi_e k_z^2\omega_i - 8is\omega_i\nu_{i/e} +$$

$$+ i\frac{\omega^2\chi_e k_z^2}{k_z^2 v_{Ti}^2}(\omega - \omega_i) + \frac{2\omega(\omega - \omega i)\nu_{i/e}\chi_e k_z^2}{k_z^2 v_{Ti}^2} = 0. \quad (2.10)$$

The unstable solution of (2.10) is

$$\operatorname{Im}\omega \sim {}^2/_9\,(8s\nu_{i/e} -$$

$$- {}^2/_3\chi_e k_z^2) \quad (\omega_i > \chi_e k_z^2 \sim \nu_{i/e} \sim k_z v_{Ti}). \quad (2.11)$$

When $\omega_i > \omega$, the existence of an instability follows from (2.9):

$$\operatorname{Im}\omega \sim \sqrt{s}\,k_z v_{Ti},$$

$$\operatorname{Re}\omega \sim k_z^2 v_{Ti}^2/\omega_i \quad (\chi_e k_z^2 \ll k_1 v_{Ti}),$$

$$\operatorname{Im}\omega \sim s v_{Ti}^2/\chi_e \quad (\chi_e k_i^2 \sim k_z v_{Ti}). \quad (2.12)$$

Instability (2.12) also exists with cold ions.

We should point out that the instabilities obtained here are poorly stabilized by crossing the field lines, since they cover the frequency range $\omega \leqslant k_z v_{T_i}$. A characteristic of the developed instability conditions is thermal expansion of the plasma.

3. Let us attempt to determine, from the known results, when the anomalous Bohm diffusion develops in a plasma, considered in the approximation of two-fluid hydrodynamics. For simplicity, we assume T_0 = const. As is well known (see [2]), if $\operatorname{Im}\omega \sim \omega_i$ and the dimension of the turbulent pulsations is on the order of the transverse dimensions of the system, the coefficient of anomalous plasma diffusion due to developed instability can be on the order of the Bohm coefficient. If $k_z v_{T_i}$ is on the order of $\omega^\circ = r^{-2}cT_0/lH_0$ (r is the characteristic transverse dimension), then, as follows from (2.12), the given instability leads to Bohm diffusion. Here we made the natural assumption that under turbulent conditions $k_x \sim k_y$, and the dimension of the turbulent pulsations can reach the transverse dimensions of the system, since the unstable conditions in question are also developed for such pulsations. Satisfaction of $\chi_e k_z^2 \leq k_z \nu_{T_i}$ is also necessary; it can be written as

$$k_z\lambda_e\sqrt{M/m} < 1, \quad (3.1)$$

where λ_e is the electron free path and m is the electron mass.

As was shown in [3], drift-dissipative instability leads to Bohm diffusion if $\omega^\circ \sim \omega_s$, where

$$\omega_s = (k_z/k_y)^2\,\omega_{Le}\,\omega_{Li}/\nu_e. \quad (3.2)$$

Here, ω_{Le} and ω_{Li} are the Larmor electron and ion frequencies, respectively, and ν_e is the electron-ion collision frequency. In this case, $k_y \sim 2\pi/r$.

Under ordinary experimental conditions, ω_s is a very large value. If we compare ω_s and $\chi_e k_z^2$, we see that the condition $\omega_s \gg \chi_e k_z^2$ gives

$$k_y v_{Ti}/\omega_{Li} \ll 1. \quad (3.3)$$

Therefore, we can conceive of the following possibilities for the development of Bohm diffusion in plasma apparatus, depending upon the parameter ω°. For sufficiently high ω_B, when $\omega w^\circ \sim \omega_s$, Bohm diffusion results from the developed drift-dissipative instability. With a decrease in ω_B, if (3.1) can be satisfied, Bohm diffusion is due to instability (2.12).

Note that the results in this paper and [3] explain, the anomalous Bohm diffusion observed in [5].

If condition (3.1) is not satisfied, Bohm diffusion can be caused by, besides drift-dissipative instability, the instability of rarefied plasma that develops at very high drift frequencies $\omega_i = k_z k_y^{-1} \cdot (M/m)^{1/2}(\omega_{Li})$ (see [1]).

In conclusion, we should point out the possibility of the following effect when field-line curvature is present. As was mentioned above, drift-dissipative instabilities can be caused by friction forces directed along H_0 which also lead to a phase shift between the forces acting on the plasma and the motion.

Let us consider a particle moving with velocity $u_\parallel$ along a magnetic field line with radius of curvature R. In a frame of reference in which the particle does not move along the field line, it is acted on by, among other things, a Coriolis force that is proportional to the angular velocity of the frame of reference $u_\parallel/R$ and the velocity of relative motion of the particle across H_0. Inasmuch as this force, like the friction force, is proportional to $u_\parallel$, it can result in similar instabilities.

Thus, in the drift approximation, in the equation for the mean longitudinal ion velocity $u_\parallel$ we have the force $f = -Mu_\parallel n_0(\mathbf{h}, (\mathbf{h}\nabla)\mathbf{w}_F$ (see [9]), where h is the unit vector along the magnetic field and w_F is the velocity of ion drift due to the force F, which has a projection onto the binormal to the field line. This force makes possible an instability with $\operatorname{Im}\omega \sim w_F/R$. If as w_F we take the velocity of electric drift, which corresponds to electric potentials in the plasma with energy on the order of the thermal energy ($w_F \sim \nu_{T_i}^2/\omega_{Li} r$), we see that anomalous particle drift with the diffusion coefficient $D \sim \sim rR^{-1}cT_0/eH_0$ is possible. For sufficiently steep toroidal systems ($r \leqslant \leqslant R$), this diffusion differs little from Bohm diffusion.

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6 June 1966 Novosibirsk